Calculus

# 4.7 L'Hospital's Rule

**Notes**

Write your questions and thoughts here!

**Recall:** When evaluating limits, first try direct substitution! $\lim\limits_{x\to 3}\frac{2x-5}{x} = \frac{1}{3}$

1. $\lim\limits_{x\to 2}\frac{x^2-7x+10}{x-2} =$

$\left.\begin{array}{l}\lim\limits_{x\to 2}(x^2-7x+10)=0\\ \lim\limits_{x\to 2}(x-2)=0\end{array}\right\}$ Indeterminate

So, $\lim\limits_{x\to 2}\frac{(x-2)(x-5)}{(x-2)}$

$\lim\limits_{x\to 2}(x-5) = \underline{-3}$

**L'Hospital's Rule:**

Suppose $f(a) = 0$ and $g(a) = 0$ and $\lim\limits_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$. L'Hopital's Rule allows you to apply the following: $\lim\limits_{x\to a}\frac{f'(x)}{g'(x)}$

Apply derivative on top & bottom

So, $\lim\limits_{x\to 2}\frac{2x-7}{1} = 4-7=-3$ } SAME PROBLEM FROM ABOVE

**Evaluate each limit. Use L'Hospital's when possible.**

2. $\lim\limits_{x\to 2}\frac{x-2}{3x^3-6x^2+x-2}$

$\lim\limits_{x\to 2}(x-2)=0 \quad \lim\limits_{x\to 2}(3x^3-6x^2+x-2)=0$

Must write → Using L'Hopital's Rule:

$\lim\limits_{x\to 2}\frac{1}{9x^2-12x+1} = \underline{\frac{1}{13}}$

3. $\lim\limits_{x\to 0}\frac{\sin(6x)}{x}$

$\lim\limits_{x\to 0}\sin(6x)=0 \quad \lim\limits_{x\to 0}x=0$

Using L'Hopital's Rule

$\lim\limits_{x\to 0}\frac{6\cos(6x)}{1} = \frac{6\cos 0}{1} = \underline{6}$

4. $\lim\limits_{x\to 0}\frac{1-\cos(x)}{x^2}$

$\lim\limits_{x\to 0}(1-\cos x)=0 \quad \lim\limits_{x\to 0}x^2=0$

Using L'Hopital's Rule

$\lim\limits_{x\to 0}\frac{\sin x}{2x}$ Still Indeterminate

$\lim\limits_{x\to 0}\frac{\cos x}{2} = \frac{1}{2}$

5. $\lim\limits_{x\to \infty}\frac{2x^2}{e^{2x}}$

$\lim\limits_{x\to\infty}2x^2=\infty \quad \lim\limits_{x\to\infty}e^{2x}=\infty$

Using L'Hopital's Rule

$\lim\limits_{x\to\infty}\frac{4x}{2e^{2x}}$

$\lim\limits_{x\to\infty}\frac{4}{4e^{2x}}=0$

CAN ALSO USE DOMINANCE

$\lim\limits_{x\to\infty}\frac{2x^2}{e^{2x}}=0$

**L'HOSPITAL'S IS NOT THE QUOTIENT RULE!!**

6. $\frac{d}{dx}\frac{\sin(6x)}{x}$

$\frac{x\,6\cos(6x)-\sin(6x)}{x^2}$ ← Derivative Using Quotient Rule

Now Try The Examples on Your Outline

| | Today's Topic: L'Hopital's Rule |
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**In-Class Examples:**

**Ex. 1** $\lim_{x\to -1}\frac{2x^2-2}{x+1}$

$\lim_{x\to -1}(2x^2-2)=0 \quad \lim_{x\to -1}(x+1)=0$

Using L'Hopital's Rule

$\lim_{x\to -1}\frac{4x}{1}=\boxed{-4}$

**Ex. 2** $\lim_{x\to\infty}\frac{3x^2-1}{2x^2+1}$

$\lim_{x\to\infty}(3x^2-1)=\infty \quad \lim_{x\to\infty}(2x^2+1)=\infty$

Using L'Hopital's Rule

$\lim_{x\to\infty}\frac{6x}{4x}=\frac{3}{2}$

**Ex. 3** $\lim_{x\to 0}\frac{e^{2x}-1}{x}$

$\lim_{x\to 0}(e^{2x}-1)=0$

$\lim_{x\to 0}x=0$

Using L'H Rule

$\lim_{x\to 0}\frac{2e^{2x}}{1}=2$

**Ex. 4** $\lim_{x\to\infty}\frac{\ln x}{x}$

$\lim_{x\to\infty}\ln x=\infty \quad \lim_{x\to\infty}x=\infty$

Using L'Hopital's Rule

$\lim_{x\to\infty}\frac{1/x}{1}=0$

**Ex. 5** $\lim_{x\to 0}\frac{\sin 5x}{\sin 3x}$

$\lim_{x\to 0}\sin 5x=0$

$\lim_{x\to 0}\sin 3x=0$

L'H Rule

$\lim_{x\to 0}\frac{5\cos 5x}{3\cos 3x}$

$\boxed{\frac{5}{3}}$

**Ex. 6** $\lim_{x\to -\infty}\frac{x^2}{e^{-x}}$

$\lim_{x\to -\infty}x^2=\infty$

$\lim_{x\to -\infty}e^{-x}=\infty$

Using L'Hopital's Rule

$\lim_{x\to -\infty}\frac{2x}{-e^{-x}}$

$\lim_{x\to -\infty}\frac{2}{e^{-x}}=0$

**AP Multiple Choice**

$\lim_{x\to 2}\frac{x^2+x-6}{x^2-4}$ is

$\lim_{x\to 2}(x^2+x-6)=0 \quad \lim_{x\to 2}(x^2-4)=0$

(A) $-\frac{1}{4}$ (B) 0 (C) 1 (D) $\frac{5}{4}$ (E) nonexistent

L'H

$\lim_{x\to 2}\frac{2x+1}{2x}$

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$\lim_{x\to 0}\frac{\sin x}{e^x-1}$ is

$\lim_{x\to 0}\sin x=0 \quad \lim_{x\to 0}(e^x-1)=0$

(A) 1 (B) $\frac{1}{e}$ (C) 0 (D) nonexistent

L'H

$\lim_{x\to 0}\frac{\cos x}{e^x}=\frac{1}{1}$

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$\lim_{x\to 3}\frac{\tan(x-3)}{3e^{x-3}-x}$ is

$\lim_{x\to 3}\tan(x-3)=0 \quad \lim_{x\to 3}(3e^{x-3}-x)=0$

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) nonexistent

L'H

$\lim_{x\to 3}\frac{\sec^2(x-3)}{3e^{x-3}-1}=\frac{1}{2}$

**Homework:** Worksheet 37